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The comparison between ad valorem and specific taxation under two-part tariffs*

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Abstract

In this paper, we compare ad valorem and specific taxation under heterogeneous demand when a monopolist offers a menu of two-part tariffs. An increase in either tax rate leads to a higher usage fee for all consumers, whereas the fixed fee under reasonable assumptions will fall. If the government changes the mix of taxes in such a way that the firm's behavior is unchanged, a system of wholly ad valorem taxation generates higher tax revenue than does a system of wholly specific taxes. Tax reform designed to leave tax revenue constant leads to a lower per usage fee and a higher fixed fee for all consumers. It also increases market coverage, profits, tax revenue, and the consumer surplus.

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1 Introduction

In the business community many firms charge consumers a single price (uniform pricing), but whenever it is feasible they will apply more sophisticated pricing strategies to increase profit. For instance, they might require consumers to pay a fee up front for the right to buy a product or service. Consumers then pay an additional fee (price) for each unit of the product they consume. Some firms charge consumers according to a single two part tariff, while some firms offer consumers a menu of tariffs and charge usage according to the individually chosen tariff. In both cases, the firm is said to apply a nonlinear pricing strategy, because the average price paid per unit depends on the total size of a consumer's purchases.¹ For example, mobile phone companies charge customers a monthly fixed fee plus a fee for message units (or calls) and offer menus of such two part tariffs. The same pricing strategy is extensively used throughout the telecommunications industry. Banks require credit card holders to pay an annual fee plus a percentage fee on the credit used. Clubs such as dating clubs, sports clubs, and golf clubs charge an annual membership fee plus a fee each time a consumer uses their facilities or services.

Despite the fact that specific (unit) taxes and ad valorem taxes are among the main revenue raisers in most OECD countries, very little is known about their effect on nonlinear pricing schemes. Nor is there any knowledge about what the optimal mix of unit and ad valorem taxes are from society's point of view when firms use nonlinear pricing. The difference between specific and ad valorem taxation under nonlinear pricing is that ad valorem taxes falls both on the fixed fee and the price per unit whereas the specific tax only falls on the quantity sold and not on the fixed fee. In this paper we compare the tax incidence and welfare effects of both type of taxes when a monopolist offers a menu of two part tariffs.

We show that a rise in either tax (specific or ad valorem) makes the price per unit go up, and that an ad valorem tax is less likely to be overshifted in the sense that

¹A two-part tariff increases the profit of a monopoly firm. Consumers are encouraged by a low per unit price to make large purchases, whereas the consumers' surpluses are extracted by the fixed fee that is paid up-front. Menus of two-part tariffs are offered because of the existence of demand-side heterogeneity. See Oi (1971), as well as Mussa and Rosen (1978) and Maskin and Riley (1984). For surveys on nonlinear pricing see Wilson (1993) and Stole (2005).

the price per unit to consumers rises by more than the tax increase. Ad valorem taxes, therefore, hurts consumers less than unit taxes. Although the sign of how a tax change affects the fixed fee cannot be uniquely determined, we find that the fixed fee most likely will fall following a rise in either tax. We also show that tax incidence under nonlinear pricing is much more complex than under uniform pricing. Under nonlinear pricing, consumers reveal their true willingness to pay only when they have the incentive to do so, i.e., if they obtain the same or larger utility by choosing the tariff intended for his demand type instead of choosing the tariff intended for a type with lower willingness to pay. Thus, if the firm increases the price for one consumer type it must also increase the price for the adjacent type to secure that all consumers continue to reveal their true type through the choice of tariff. We find that under nonlinear pricing a tax can be shifted differently across consumers and a tax change affects the whole set of menus offered by the firm. This is in contrast to uniform pricing where the issue of whether a tax is over- or undershifted depends on the curvature of the demand function.

A second set of results pertains to the welfare and tax revenue effects of specific and ad valorem taxes. If the government changes the mix of taxes so that the firm's behavior is unchanged, a pure system of ad valorem taxation generates higher tax revenue than does a pure system of specific taxation. Ad valorem taxes, therefore are more efficient in raising tax revenue. Furthermore, a tax reform that is designed to leave tax revenues at the initial tariff menu unchanged, leads to a lower price per unit, but a higher fixed fee for all consumers. Such a reform also increases market coverage, yields higher profit and generates a larger consumer surplus, hence, it must also generate higher tax revenue. The policy insight from such a reform, then, is that the ad valorem tax strictly Pareto dominates the specific tax under a menu of two-part tariffs.

In order to bring forward these results we study a monopolist firm which supplies a single good to consumers who are identical except for their marginal willingness to pay for the monopolist's product. The firm's problem is to design a menu of two-part tariffs (each consisting of a fixed fee and a price per unit), such that all consumers find it individually rational to select the tariff that is intended for his or her type, given that individual willingness to pay is private information

to the consumer.² Real world examples of nonlinear pricing by means of two part tariffs are often characterized by firms offering consumers a limited menu of two part tariffs. In our model we assume that the monopoly firm offers a continuum of two part tariffs. This simplifies the analysis but yields qualitatively the same results as if the model contained a discrete number of tariffs.

Our analysis relates to a substantial literature that compares specific taxes to ad valorem taxes under uniform pricing.³ Suits and Musgrave (1953) finds that ad valorem taxation yields a larger total surplus than unit taxes provided they give the same yield. Skeath and Trandel (1994) shows that ad valorem taxes Pareto dominate specific taxes. More recently, Delipalla and Keen (1992) compare ad valorem to specific taxes in models of oligopoly and show that ad valorem taxes imply a lower consumer price, higher tax revenue, and lower profits (if entry is precluded) than specific taxes. All these studies are undertaken in a framework where a firm is charging consumers a single price (linear pricing).

Studies that compare unit taxes and ad valorem taxes under nonlinear pricing are scant. Damus (1981) finds that taxation distorts the profit maximizing behavior of firms using two-part tariffs. His analysis does not make an attempt to distinguish and compare unit and ad valorem taxation nor to study tax incidence. Cheung (1998) compares ad valorem to unit taxes examining first-, second, and third degree price discrimination, and finds that under any of these pricing schemes ad valorem taxes Pareto dominates unit taxes. The price structure in Cheung (1998) represents average prices and his focus is on the direction of output changes following a tax change. Our analysis distinguishes itself from the two above in that it uses a general two part pricing model to examine and compare tax incidence and welfare effects under ad valorem and unit taxes.

In Section 2, we outline the basic model that incorporates indirect taxation. In Section 3, we focus on the isolated effects of a change in either the ad valorem tax or the specific tax on the individual fixed fee and the price per unit. In Section

²In standard mechanism design theory the constraints on the firm's maximization problem are referred to as the incentive compatibility constraint and the participation constraint of each consumer. See e.g. Fudenberg and Tirole (1991, ch. 7).

³Comparison of ad valorem and unit taxes dates back to Cournot (1960) and Wicksell (1959). More recently Suits and Musgrave (1953), Cournot (1960) and Wicksell (1959) study indirect taxation and tax incidence under monopoly. A survey of the tax incidence literature is given in Fullerton and Metcalf (2002) whilst Keen (1998) surveys specific and ad valorem taxation.

4, we examine an output-neutral shift from specific to ad valorem taxation and we investigate how this affects tax revenues. In Section 5, we investigate the effects on tariff menu and welfare of a tax-revenue-neutral shift from specific to ad valorem taxation. In Section 6 we offer some concluding remarks.

2 Nonlinear pricing with indirect taxation

The model we use is one with a monopoly firm producing and selling a single good at constant marginal cost c to many consumers. The consumers differ in their willingness to pay for the good and their differences in taste are defined by a single dimensional parameter θ , which can be interpreted as a quantity-type parameter (the higher is θ , the larger is demand at any given tariff). The monopoly firm offers a menu of two-part tariffs to the consumers, where p is the unit price of the good and A denotes the fixed fee a consumer must pay in order to purchase the desired quantity q . We study a Bayesian game in which the monopolist first chooses a menu of two-part tariffs. Each consumer subsequently selects at most one tariff from the menu. If the consumer selects a tariff $\{p, A\}$, he or she pays $pq + A$ for q units of the good. It should be made clear at the outset that we will consider a continuum of two-part tariffs and we shall therefore use the terms two-part tariffs and nonlinear pricing interchangeably.

A consumer derives utility according to the quasilinear utility function

$$U = \begin{cases} u(q, \theta) - pq - A, & \text{if } q > 0 \\ 0, & \text{otherwise,} \end{cases}$$

where $u(q, \theta)$ is the gross surplus and $pq + A$ is the monetary transfer from the consumer to the monopolist.

We assume that $u(q, \theta)$ is increasing and concave in q for finite values of q , that $u(0, \theta) = 0$ and that it is increasing in θ for all values of θ . We impose the standard Single Crossing condition that prevents the demand curves of two different types of consumer from crossing. This amounts to assuming that $u_{q\theta} > 0$. Furthermore, in order to ensure the existence of a unique solution for consumers' choices $q(p, \theta)$ for a per unit price equal to marginal cost we use the standard assumptions that

$$u_q(\infty, \theta) \equiv \lim_{q \rightarrow \infty} u_q(q, \theta) \leq 0 \text{ and } u_q(0, \theta) \equiv \lim_{q \rightarrow 0} u_q(q, \theta) \gg c.^4$$

Since the utility function is quasilinear, the demand function maximizes the consumer surplus, and the area under the demand curve is a consumer's gross surplus measured in monetary terms. High demand types have larger consumer surplus for a given per usage fee than low demand types, indicating that the firm can charge them a higher fixed fee. The monopolist has prior beliefs about the distribution of types, $\theta \in [\underline{\theta}, \bar{\theta}]$. This distribution is described by a cumulative distribution function $F(\theta)$ that is differentiable. The corresponding density function, $f(\theta)$, is strictly positive on the support.

A consumer of type θ who maximizes utility subject to a tariff of $T = pq + A$, chooses an optimal amount equal to $q(p, \theta)$, and receives indirect utility

$$v(p, \theta) - A = \int_p^\infty q(z, \theta) dz - A.$$

The monopolist wishes to separate consumers whose willingness to pay differs by offering a continuous menu of two-part tariffs given by $\{p(\theta), A(\theta)\}_{\theta \in [\theta^*, \bar{\theta}]}$, where $[\theta^*, \bar{\theta}]$ is the market coverage of the firm, and θ^* denotes the consumer that is just indifferent between making a purchase or not.⁵ If the monopolist serves the whole market (all types) then $\theta^* = \underline{\theta}$. Otherwise the last consumer being served (cut-off type) is $\theta^* > \underline{\theta}$.

The tariff menu $\{p(\theta), A(\theta)\}$ must be designed such that each type θ chooses the tariff intended for his or her type, and such that each type $\theta \geq \theta^*$ finds it individually rational to accept the tariff rather than not participate and receive the reservation utility of zero. Hence, the firm maximizes profit subject to a set of incentive compatibility constraints and a set of individual rationality constraints (participation constraints). In equilibrium, a type- θ consumer chooses the tariff intended for him. Let $V(\theta, \theta) \equiv V(\theta)$ denote the equilibrium utility level he or

⁴The canonical version of the simple model that is presented in this paper can be found in Tirole (1988, ch 3.5), and in Fudenberg and Tirole (1991, ch 7). See Wilson (1993) on nonlinear pricing by a monopolist; see Rochet and Stole (2003) for a guide to the screening literature; and see Stole (2005) for a comprehensive guide to the literature on price discrimination in models that incorporate competition.

⁵Note that the tariff menu will depend not only on θ , but also on the taxes in question. We will introduce these parameters when we characterize the optimal tariff menu.

she enjoys when he or she chooses the tariff intended for him or her; that is,

$$V(\theta) = v(p(\theta), \theta) - A(\theta) \quad (1)$$

When a consumer of type θ chooses some arbitrary tariff $\{p(\theta'), A(\theta')\}$, the net utility is $V(\theta, \theta'), \theta) - A(\theta')$. Thus, for all θ , the constraints are

$$\begin{aligned} v(p(\theta), \theta) - A(\theta) &\geq 0, \\ v(p(\theta), \theta) - A(\theta) &\geq v(p(\theta'), \theta) - A(\theta'). \end{aligned}$$

Given our assumptions, it is well known from the mechanism design literature that the full set of participation and incentive constraints can be replaced by the following two constraints:

$$A(\theta) = v(p(\theta), \theta) - V(\theta), \quad \forall \theta \in [\theta^*, \bar{\theta}] \quad (2)$$

$$V(\theta) = \int_{\theta^*}^{\theta} v_{\theta}(p(u), u) du, \quad \forall \theta \in [\theta^*, \bar{\theta}] \quad (3)$$

together with $p(\theta)$ being monotonically nonincreasing over the type space.⁶ Since the incentive constraint requires that $-q(p(\theta), \theta)p'(\theta) = A'(\theta)$ we know that the fixed fee must be nondecreasing over the type space. When the firm offers the menu $\{p(\theta), A(\theta)\}$, the profit function is

$$\Pi = \int_{\theta^*}^{\bar{\theta}} \left[((1 - t_v)p(\theta) - t_s - c)q(p(\theta), \theta) + (1 - t_v)A(\theta) \right] f(\theta) d\theta, \quad (4)$$

where t_v is an ad valorem tax, t_s a specific tax, θ^* is the cut-off type, and $\{p(\theta), A(\theta)\}$ satisfies equations (2) and (3).

It should be made clear that we follow the conventional definition of specific taxes and value added taxes in that the former falls on what constitutes one unit of the good sold whereas the value added tax falls on the total value of the transaction undertaken. Specific taxes are taxes on special characteristics of commodities (here volume) leaving untaxed some characteristics of the good

⁶This is a standard result in these types of models, and a proof can be found in Fudenberg and Tirole (1991, chapter 7) or Tirole (1988, chapter 3.5)). A sketch is given in Appendix A.

(such as the pleasures of an amusement park, say); ad valorem taxes, in contrast, fully taxes the whole set of characteristics of a good. Therefore, ad valorem taxes fall on both the fixed fee and the price per unit whereas the specific tax only falls on the quantity sold, since the fixed fee is the price the consumer has to pay in order to enter the market and is not quantity related. Obviously we could have allowed hybrid tax systems where the ad valorem tax falls on the fixed fee and the specific tax on the number of transactions, but we do not do this since the purpose at hand here is to investigate these two tax schemes in the same spirit as previous studies under uniform pricing.

Substituting (2) and (3) into (4) and integrating by parts, we can rewrite the profit function as

$$\Pi = \int_{\theta^*}^{\bar{\theta}} (1 - t_v) \left[(p(\theta) - \phi)q(p(\theta), \theta) + v(p(\theta), \theta) - \frac{1-F(\theta)}{f(\theta)}v_\theta(p(\theta), \theta) \right] f(\theta) d\theta, \quad (5)$$

where $\phi \equiv (t_s + c) / (1 - t_v)$ is the effective marginal cost per unit. The term under the integrand is the firm's 'virtual profit' and is defined as $\pi(p(\theta), \theta, t_s, t_v)$.

Maximization of (5) requires pointwise maximization for each consumer type (θ) and yields the following pricing rule:⁷

$$p(\theta) = \phi + \frac{1 - F(\theta)}{f(\theta)} \left(\frac{v_{\theta p}}{-v_{pp}} \right). \quad (6)$$

The solution in p to equation (6) gives the price per unit $p = p(\theta, t_s, t_v)$. For each type, this should be equal to the effective marginal cost plus a correction term. The correction term is the inverse hazard rate of $F(\theta)$ multiplied by a fraction that represents the trade-off between the informational rent and the consumer's marginal willingness to pay the entrance fee (that is, $(v_{\theta p} / -v_{pp})$).

⁷As is standard, we assume that the second-order conditions for this maximization problem are satisfied. The second-order condition for global incentive compatibility is that the per unit charge is decreasing in θ . This condition is satisfied when the firm's marginal profit is decreasing in θ (that is, when $\partial^2 \pi(p(\theta), \theta, t_s, t_v) / \partial p \partial \theta < 0$). One necessary condition for this is that the distribution $F(\theta)$ satisfies the monotone hazard rate condition, which is a standard assumption in the nonlinear pricing literature. It specifies that the hazard rate of the distribution, $\frac{f(\theta)}{1-F(\theta)}$, is increasing in θ , and that the inverse hazard rate, $\frac{1-F(\theta)}{f(\theta)}$, is not increasing.

The numerator, $(v_{\theta p})$, indicates how the price per unit affects the information rent for the type that is just above θ , whereas the denominator, $(-v_{pp})$, measures how the price per unit affects the surplus of consumer type θ . The full correction term is therefore the marginal cost of revealing private information held by the consumer, which is the cost of screening.

An alternative way of expressing the pricing rule is to use a variant of the Lerner index of monopoly power, as follows:

$$\frac{p - \phi}{p} = \left[\frac{1 - F(\theta)}{\theta f(\theta)} \right] \frac{E_{\theta}}{E},$$

where $E_{\theta} = q_{\theta}(\theta/q) > 0$ and $E = -q_p(p/q) > 0$ are the elasticities of demand with respect to θ and p , respectively. The Lerner index under uniform pricing is given by $(p^{UP} - \phi)/p^{UP} = 1/\hat{E}$ where \hat{E} is the elasticity of demand in terms of aggregate demand. Different from the standard interpretation of the Lerner index under uniform pricing is the presence of the demand elasticity with respect to consumer type, weighted by the price elasticity. As in standard theory with uniform pricing the more price sensitive the consumers, the lower is the mark-up over marginal cost. On the other hand, the larger the elasticity of demand with respect to consumer type at a given price, the larger is the mark-up.

If the firm has perfect information about each consumer's valuation there is no information rent to consider and the last term in the bracket in (5) vanishes and $p = \phi$ for $\bar{\theta}$. If consumer valuations are private information, it follows from equation (6) that $p > \phi$ for every consumer except for the one with the highest demand ($F(\bar{\theta}) = 1$). The correction term, then, shows that the informational cost pertaining to any given consumer θ is higher for low-demand types under the assumption that the inverse hazard rate is nonincreasing in θ . This means that the firm may profit by distorting the price per unit to the extent that it excludes some low-demand types from the market. When the solution to (6) implies negative outcomes with respect to individual demands, $q(p(\theta, t_s, t_v), \theta)$, these types are not served. The critical value for market coverage is

$$q(p(\theta^*, t_s, t_v), \theta^*) = 0. \quad (7)$$

Hence, the firm excludes consumers in the interval $\theta \in [\underline{\theta}, \theta^*]$.

3 Tax incidence under nonlinear pricing

In this section, we focus on how a change in either the ad valorem tax or the specific tax affects firm behavior, as well as producer and consumer surplus.

For clarity of exposition, we let the elasticity of the slope $q_p(p, \theta)$ with respect to p be defined by ε_p , and let the elasticity of the slope $q_\theta(p, \theta)$ with respect to p be defined by ε_θ . Concerning the second order derivative of the demand function we assume that it is convex, that is $q_{pp} \geq 0$, whereas $q_{p\theta} \gtrless 0$. Hence, we define

$$\varepsilon_p(p, \theta) \equiv \frac{q_{pp}p}{q_p} \leq 0, \quad \varepsilon_\theta(p, \theta) \equiv \frac{q_{p\theta}p}{q_\theta} \gtrless 0.$$

An important issue in what follows is who bears the economic burden of the tax. Is the tax passed on to the consumer or the producer or is it shared between them? The standard definition in the literature on tax incidence is that a specific tax is ‘overshifted’ if $dp/dt_s > 1$ in the absence of a preexisting ad valorem tax, whereas an ad valorem tax is overshifted if $dp/dt_v > p$; that is, if the percentage increase in the price exceeds the percentage rise in the tax. Similarly, we use the term ‘undershifted’ to describe $dp/dt_s < 1$ and $dp/dt_v < p$. In what follows, we concentrate on how the fee structures across types of consumer are affected by changes in either tax.

In order to see how the incentives for tax shifting onto the per unit price differ under uniform pricing and a two-part tariff, we differentiate equation (6) and obtain the expression measuring the tax incidence under specific taxation as follows:

$$\frac{dp}{dt_s} = \frac{d\phi}{dt_s} \left[1 + L(\theta)(\varepsilon_p - \varepsilon_\theta) \right]^{-1}, \quad (8)$$

where $L(\theta)$ is the Lerner index under nonlinear pricing as defined in Section 2. Note that the concavity of the profit function prevents dp/dt_s from being negative. If the term in the squared bracket in (8) is larger than unity, t_s is overshifted. The question of overshifting or undershifting, thus, depends entirely on the size of ε_p and ε_θ , whereas the size of the shift depends on the mark-up as well.

For comparison, the effect of a change in the specific tax on the usage fee under

uniform pricing is given by

$$\frac{dp^{UP}}{dt_s} = \frac{d\phi}{dt_s} \left[1 + L^{UP} (\hat{E} + \hat{\varepsilon}_p) \right]^{-1}, \quad (9)$$

where $\hat{\varepsilon}_p$ and \hat{E} are defined in terms of aggregate demand rather than in terms of type-dependent demand functions, and L^{UP} is the (constant across all individuals) Lerner index under uniform pricing. The condition for overshifting under monopoly and uniform pricing (all consumers face the same price) are well known in the public finance literature and can be summarized as follows

Lemma 1 *Under uniform pricing, concave demand will always give undershifting, whereas overshifting occurs if the demand function is sufficiently convex, that is, if the elasticity of the demand curve (\hat{E}) at p^{UP} is lower than minus the elasticity of the slope of the demand curve ($\hat{\varepsilon}_p$) at p^{UP} .⁸*

Under a menu of two part tariffs there is in addition to the curvature of the demand function the issue of incentive compatibility to take into account when analyzing conditions for tax incidence. Recall that the individuals are charged a type dependent per unit charge $p(\theta)$, which is set to maximize the revenue net of cost for each individual minus the informational rent required to induce the consumer to choose the tariff intended for him. Therefore, if the monopolist increases the price for one consumer of type θ , it has to increase the price for the adjacent type just below θ as well in order to restore the incentive compatibility constraint. Hence, the monopolist will not only take into account that an increase in the per unit charge $p(\theta)$ changes this type's demand, but also that the demand for the adjacent type will change as well. These effects are captured by ε_p and ε_θ respectively.

Equation (8) states the tax incidence under non-linear pricing and specific taxation. Accordingly, the incidence of the ad valorem tax under non-linear pricing can be written as

$$\frac{dp}{dt_v} = \phi \frac{dp}{dt_s}. \quad (10)$$

⁸Note that the restriction on the slope of the demand curve ($\hat{\varepsilon}_p$) takes into account the curvature of the demand curve.

The discussion above has made it clear that the effect of a change in either tax depends can be summarized as follows:

Proposition 1 *The price per unit $p(\theta, t_s, t_v)$ increases for all consumers following an increase in either t_v or t_s .*

- (a) *For the consumer who is most willing to pay (has the highest θ), a one-percent increase in the tax is shifted onto the consumer by a one-percent increase in the price per unit.*
- (b) *For consumer types in $[\theta^*, \bar{\theta})$, an increase in t_s is overshifted if $\varepsilon_\theta(p, \theta) > \varepsilon_p(p, \theta)$, and undershifted if $\varepsilon_\theta(p, \theta) < \varepsilon_p(p, \theta)$.*
- (c) *If an ad valorem tax is overshifted, then a specific tax will also be overshifted. The converse, however, is not true.*

Proof. See Appendix B for a proof of Proposition 1.

Equations (8) and (10) as well as result (a) and (b) in Proposition 1 show very clearly that the tax incidence conditions differ from those under uniform pricing and now relate to the combined effects of incentive compatibility and curvature of demand characteristics.

In order to further make clear the difference between the two pricing schemes, note from (8) that under non-linear pricing a specific tax is overshifted if the elasticity with respect to p of the slope q_θ is larger than the elasticity with respect to p of the slope q_θ ($\varepsilon_\theta > \varepsilon_p$). This differs from uniform pricing, where a specific tax is overshifted if the demand curve is sufficiently convex (otherwise it is undershifted). Furthermore, it is well known under uniform pricing that a linear demand function implies that 50% of a specific tax increase is passed on to the consumer price. In contrast, linear demand under nonlinear pricing means that a specific tax is overshifted if $q_{p\theta} > 0$, i.e., if the slope of the demand function is steeper for consumer types with larger willingness to pay, i.e., for larger θ . Moreover, if the slope of the demand curve is the same across types, a tax increase is always overshifted for convex demand, and always undershifted for concave demand. Result (c) is, however, in line with findings of tax incidence

under uniform pricing both under monopoly and oligopoly models (see Delipalla and Keen (1992)).

Proposition 1 has made clear the incidence on the unit price (p), and we now turn to examine the effects of taxation on the fixed fee (A). The tax incidence for the specific and ad valorem tax, respectively, are given by (the full derivation is relegated to Appendix C)

$$\frac{dA}{dt_s} = -q(p(\theta, t_s, t_v), \theta) \frac{dp}{dt_s} - \int_{\theta^*}^{\theta} v_{\theta p}(p(u, t_s, t_v), u) \frac{dp}{dt_s} du, \quad (11)$$

and

$$\frac{dA}{dt_v} = \phi \frac{dA}{dt_s}. \quad (12)$$

The effect of an increase in either tax rate on the fixed fee can be decomposed into its impact on the consumer's gross surplus and its effect on the information rent. The gross surplus falls following a rise in the tax rate and, ceteris paribus, this suggest that A should fall. However, the rise in either tax rate also reduces the information rent and this effect taken in isolation suggests that A should go up. The relative magnitudes of these two effects, then, are opposing. From equation (11) it is clear that the fixed fee is nonincreasing for the very lowest type, θ^* . The change in the incidence on the fixed fee for a $\theta > \theta^*$ is determined by

$$(i) \quad \frac{d^2 A}{dt_s d\theta} = - \left[q_p \frac{dp}{dt_s} \frac{dp}{d\theta} \right] - \left[q \frac{d^2 p}{dt_s d\theta} \right], \quad (ii) \quad \frac{d^2 A}{dt_v d\theta} = \phi \frac{d^2 A}{dt_s d\theta}. \quad (13)$$

If $d^2 A / dt_s d\theta < 0$ over the entire type space, the fixed fee is nonincreasing for all types above θ^* . The first squared bracket in (i) is positive from Proposition 1 and the global incentive compatibility condition so the sign of $d^2 A / dt_s d\theta$ depends of the sign and magnitude of the second bracketed term in (i). In the remainder of this section we assume for simplicity that θ is uniform on a unit length interval. This assumption does not affect the results to follow, but simplifies the calculations leading to them. By rewriting expression 13(i) it can be shown that $d^2 A / dt_s d\theta \leq 0$ if

$$\left(2 \frac{dp}{dt_s} + \frac{E}{\varepsilon_p} - 1 \right) \frac{dp}{d\theta} + \frac{q_{p\theta}}{q_{pp}} \geq 0 \quad (14)$$

We may now state the following proposition

Proposition 2 *The effect on the fixed fee is summarized as follows:*

- (a) *The fixed fee is always nonincreasing in t_s and t_v for the consumer that is just indifferent between making a purchase or not (θ^*).*
- (b) *If a specific tax is undershifted onto the per unit price, the fixed fee is nonincreasing for the consumer with the highest demand ($\bar{\theta}$).*
- (c) *If the demand curve is linear ($q_{pp} = 0$) or if $\frac{dp}{dt_s} \leq \frac{1}{2}$, the fixed fee $A(\theta, t_s, t_v)$ is strictly decreasing in t_s and t_v for all $\theta \in (\theta^*, \bar{\theta}]$.*
- (d) *If $\frac{dp}{dt_s} \geq \frac{1}{2}$ and the demand curve is convex, the result in (c) holds provided that $\frac{E}{-\varepsilon_p}$ is sufficiently large.*

Part (a) of Proposition 2 can be verified by inspection of (11) and (12). Part (b)-(d) is proved in Appendix C.

Proposition 2 shows that the effect on consumer surplus dominates the information rent effect for a large set of demand specifications. Consequently, the fixed fee falls in order to restore the participation constraint. Any change that goes beyond the scope of restoring the participation constraint cannot be optimal because such changes imply that profit was not maximized at the initial level. In principle, the firm could shift a tax increase onto both the usage fee and the fixed fee, but it chooses in most cases instead to increase the usage fee and reduce the fixed fee.⁹ The reason is that the firm has two instruments at its disposal. The primary role of the price per unit is to separate consumers, whereas the fixed fee is an instrument that is used to extract the residual consumer surplus subject to the participation and incentive constraints. The first-order condition (6) shows that a rise in either tax rate increases the effective marginal cost of the firm and makes the firm increase its usage fee.

⁹This is evident from inspection of equation (14) and includes for instance all quadratic and log-linear utility functions.

4 An output-neutral shift from specific to ad valorem taxation

The purpose of this section is to compare how tax revenue changes if the government switches from a wholly ad valorem tax system to a system of wholly specific taxation, given that the allocation of the good or service across consumers is the same under both tax systems (i.e., the per unit charges and fixed fees are kept constant). It is well known in the public finance literature that for any given specific tax t_s , there exists some ad valorem tax t_v such that the firm's profit, output, and the consumer surplus are equal. In our case the pricing rule implied by (6) shows that the allocation of the good or service across consumers is the same under specific and ad valorem taxation if

$$c + t_s = \frac{c}{1 - t_v}.$$

This expression corresponds to an output-neutral tax mix under uniform pricing.

Let tax revenues be R_V and R_S , respectively, under a pure ad valorem tax system (wholly ad valorem tax) and under a pure specific tax system (wholly specific tax). We use the superscripts TP and UP to denote tax revenues under a menu of two-part tariffs and under uniform prices respectively. A comparison of the tax revenues under wholly ad valorem and wholly specific taxation yields the following proposition.

Proposition 3 *A wholly ad valorem tax system that generates the same price per unit and fixed fee profiles as does a wholly specific tax system generates higher tax revenue. Furthermore, maximum tax revenue is higher under a menu of two-part tariffs than under a uniform price; that is,*

$$R_V^{TP} - R_S^{TP} > R_V^{UP} - R_S^{UP} > 0.$$

Proof. The full set of calculations is given in Appendix D. A proof of $R_V^{UP} - R_S^{UP} > 0$ is given in Suits and Musgrave (1953) and Skeath and Trandel (1994).

In the Appendix, we show that

$$R_V^{UP} - R_S^{UP} = t_v \Pi^{UP}, \quad (15)$$

where Π^{UP} is maximized profit under uniform pricing, and

$$R_V^{TP} - R_S^{TP} = t_v \Pi^{TP}, \quad (16)$$

where Π^{TP} is maximized profit under nonlinear pricing. It follows from the profit maximization hypothesis that if uniform pricing is an available option for the firm, it will use a menu of two part tariffs only when this generates higher profit than uniform pricing. Thus, when nonlinear pricing is used, it follows that $(R_V^{TP} - R_S^{TP}) = t_v \Pi^{TP} > t_v \Pi^{UP} = (R_V^{UP} - R_S^{UP})$. ■

Proposition 3 makes it clear that targeted tax revenue can be attained with lower per unit charges for every consumer under ad valorem taxation compared to specific taxation. From this conclusion also follows the insight that market coverage is larger under a wholly ad valorem tax, and that a nonlinear pricing scheme is more efficient in terms of extracting consumer surplus and leads to higher profit and higher tax revenue than does uniform pricing. From the perspective of the government, the ad valorem tax, therefore, is to be preferred since it is more efficient at raising tax revenue.

5 A tax-revenue-neutral shift from specific to ad valorem taxation

Given that a wholly ad valorem system of taxation is preferable to wholly specific tax system when tax revenue is concerned, a natural follow up question is if welfare can be increased by placing more emphasis on the ad valorem tax in a system of mixed taxation. We follow closely the analysis in Delipalla and Keen (1992) and focus on a tax reform that, while not fully tax revenue neutral in general, has no “first round” effect on tax revenue. In particular, if the firm implements the tariff menu $\{p(\theta, t_s, t_v), A(\theta, t_s, t_v)\}$, which implies market coverage of $[\theta^*, \bar{\theta}]$,

tax revenue is

$$R = \int_{\theta^*}^{\bar{\theta}} \left[(t_v p(\theta, t_s, t_v) + t_s) q(p(\theta, t_s, t_v), \theta) + t_v A(\theta, t_s, t_v) \right] f(\theta) d\theta.$$

Thus, tax revenue is given by $R(t_s, t_v) = R(t_s, t_v, p(\theta, t_s, t_v), A(\theta, t_s, t_v), \theta^*(t_s, t_v))$, and a reform with no first-round effect on tax revenue at the initial tariff menu satisfies the condition that

$$dR(t_s, t_v) = \frac{dR}{dt_s} dt_s + \frac{dR}{dt_v} dt_v = 0 \quad \Leftrightarrow \quad -dt_s = \frac{dR/dt_v}{dR/dt_s} dt_v.$$

To be more precise, the tax reform is given by the rule

$$\bar{p} dt_v = -dt_s \tag{17}$$

where \bar{p} is the average revenue per unit across total production.

The reform in question, then, is one which alters the mix of taxes by tilting the balance towards ad valorem taxation. As shown in Propositions 1 and 2, the firm responds to the change in tax mix by altering its tariff menu. The real effects of such a reform are:

Proposition 4 *A tax reform that shifts taxation from specific to ad valorem tax, but has no first-round effects on tax revenues has the effects that:*

- (a) *It lowers the price per unit and increases the fixed fee for all consumers;*
- (b) *It increases market coverage, the consumer surplus, and tax revenues;*
- (c) *It has a neutral effect on the firm's profit.*

It follows from Proposition 4 that we may state:

Corollary 1 *An ad valorem tax weakly Pareto dominates a specific tax.*

Proof. See Appendix E for a proof of Proposition 4 and Corollary 1.

Proposition 4 shows that changes in indirect taxes affect a monopoly using a nonlinear price scheme qualitatively in the same way as a monopoly setting a uniform price. The mechanisms at play are well known from previous studies and a discussion of these is therefore omitted here (see e.g. Skeath and Trandel (1994)). The preference for ad valorem taxation under a uniform price is known to be due to the multiplier effect. Because a price increase raises government tax revenue, a targeted one-percent increase in the producer price implies that the consumer price must rise by more than one percent. To see this, let p_n be the price received by the producer and let p be the consumer price per unit. Then, $p_n = (1 - t_v)p - t_s$. Totally differentiating this relationship with respect to p_n and p yields

$$\frac{dp}{dp_n} = 1/(1 - t_v) > 1.$$

Hence, a firm that wants to increase its producer price by one percent must increase the price charged to consumers by more than one percent ($1/(1 - t_v) > 1$). If there is no ad valorem tax but only a specific tax, there is a one-to-one relationship between the increases in the producer and consumer prices.

The multiplier effect under nonlinear pricing must account for the fact that a change in the per unit charge in a given consumer's tariff must be followed by a change in the fixed fee in order to satisfy the incentive constraint. For a given consumer type θ , revenue per unit to the producer, $(\bar{p}_n(\theta))$, consists of the usage fee and the fixed fee as follows:

$$\bar{p}_n(\theta) = (1 - t_v)p(\theta) - t_s + (1 - t_v)\alpha(p(\theta), \theta)\frac{v(p(\theta), \theta)}{q(p(\theta), \theta)},$$

where $0 \leq \alpha(p(\theta), \theta) \leq 1$ is a proxy for the monopolist's ability to capture the surplus of the consumer, $v(p(\theta), \theta)$, by use of the fixed fee. We can show that

$$\frac{dp(\theta)}{d\bar{p}_n(\theta)} = \frac{1}{1 - t_v} \left[\frac{1}{1 - \alpha + \frac{E}{\varepsilon_v} \left(\alpha'_p \left(\frac{q}{-q_p} \right) + \alpha \right)} \right] \equiv \frac{1}{1 - t_v} \Phi(p(\theta), \theta), \quad (18)$$

where $E \equiv -q_p p/q$ is the price elasticity of demand and $\varepsilon_v \equiv -v_p p/v$ is the elasticity of the consumer surplus with respect to p . Equation (18) shows that the

multiplier effect under nonlinear pricing is equal to that under uniform pricing, namely $1/(1 - t_v)$. Although Φ varies over the type space, the presence of an ad valorem tax $t_v > 0$ creates a multiplier effect also under nonlinear pricing, which is the driving force for the difference between ad valorem and specific taxation.

6 Conclusion

The focal point in this paper has been on how indirect taxation affects a monopoly firm that uses a nonlinear pricing scheme, and where the willingness to pay for the product sold is private information to the consumer. It is well known that the firm's optimal response to such information asymmetry is to introduce a price-cost distortion in the per unit price to balance the trade-off between extracting consumer surplus from low-demand consumers through lower per unit charges, and extracting informational rents from high-demand consumers through higher fixed fees. In the paper we show how indirect taxation affects this trade-off. We show that the price per unit rises following a tax increase (specific or ad valorem), and that the rise in the unit price differs substantially across consumers depending on their willingness to pay for the good sold. The effect on the fixed fee of a change in either tax rate is in general ambiguous, but for plausible assumptions (such as quadratic and log-linear utility functions, for example), the fixed fee will fall. We find tax incidence under nonlinear pricing to be more complex than under uniform pricing. In the latter case the shape of the demand function determines if a tax is over- or undershifted, say. Under nonlinear pricing, the incidence analysis is more complex. The reason is that a change in the per unit charge towards a specific consumer interferes with her incentive to reveal her true willingness to pay, unless there is a simultaneous change in the fixed fee or in the tariff offered to an adjacent type. In response, a monopoly firm will change both the fixed fee that is charged to her, as well as the per per unit price that is charged to the adjacent consumer type. Thus, if the firm increases the price for one consumer type it must also increase the price for the adjacent type to secure that all consumers continue to reveal their true type through the choice of tariff.

Our study also shows that the presumption in favor of ad valorem taxation under linear pricing extends to nonlinear pricing. If the government changes the mix

of taxes without the behavior of the firm being affected, a wholly ad valorem taxation system generates higher tax revenue than does a system of wholly specific taxes. Furthermore, a tax reform that places more emphasis on ad valorem taxation and does not have first round effects on tax revenue, leads to a lower price per unit and (most likely) increases the fixed fee for all consumers. Such a reform broadens market coverage, increases profits, tax revenue, and consumer surplus. These effects are greater under ad valorem taxation, so the ad valorem tax Pareto dominates a specific tax.

Our results may not be robust to changes in the characterization of imperfect competition, although it is a fact that the literature on uniform pricing finds that the presumption in favor of the ad valorem tax is still valid (see Delipalla and Keen (1992)). A further issue that has been omitted here is the choice of product quality. As shown by Kay and Keen (1983, 1991), the optimal balance between ad valorem and specific taxes then depends on the precise form of consumer preferences. This may well be the case in a setting of non-linear pricing, but additional conditions may apply as well.

Appendix

A Implementable two-part tariffs

Instead of assuming that consumers choose a tariff, consider the case in which a consumer announces a type θ' and is offered a tariff contingent on this announcement, $\{p(\theta'), A(\theta')\}$. Given this mechanism, a consumer of type θ maximizes utility with respect to a type announcement.

$$\theta \in \arg \max_{\theta'} \{v(p(\theta'), \theta) - A(\theta')\}.$$

Hence, $-qp'(\theta) = A'(\theta)$ and there must be an inverse relationship between $p(\theta)$ and $A(\theta)$, in which case the firm can increase the fixed fee if the per usage fee is decreased. The mechanism is locally incentive compatible if a consumer type θ is not tempted to report a type marginal below his or her true type. The local incentive compatibility constraint is derived by applying the envelope theorem as follows:

$$\frac{\partial V}{\partial \theta} = v_{\theta}(p(\theta), \theta).$$

The second-order condition for the choice of report θ' is that $V_{\theta'\theta'}(\theta, \theta) < 0$. Differentiating the first-order condition $V_{\theta'}(\theta, \theta) = 0$ with respect to θ yields $V_{\theta'\theta'}(\theta, \theta) = -V_{\theta'\theta}(\theta, \theta)$. Hence, a sufficient condition for global incentive compatibility is that $-q(p(\theta), \theta) \frac{dp}{d\theta} > 0$. Consequently, for a tariff menu to be implementable, $p(\theta)$ must be decreasing and $A(\theta)$ must be increasing. Integrating the local incentive constraint yields

$$V(\theta) = V(\theta^*) + \int_{\theta^*}^{\theta} v_{\theta}(p(u), u) du.$$

When $V(\theta^*) = 0$, the participation and incentive constraints in equations (2) and (3), together with $p'(\theta) < 0$, guarantee that the constraints are satisfied globally as well as locally. The monotonicity condition, $p'(\theta) < 0$, is ignored in the optimization; instead we must check that it is satisfied.

B Proof of Proposition 1

Proposition 1 can be verified by differentiating the first-order condition $\pi_p(p(\theta, t_s, t_v), \theta, t_s, t_v) = 0$ (that is, (6)) with respect to p and t_s to verify parts (a) and (b), and with respect to p and t_v to verify part (c). We find that

$$\frac{dp}{dt_s} = q_p \frac{1}{\pi_{pp}} > 0.$$

Because the firm's marginal cost is given by $\phi = (c + t_s)/(1 - t_v)$, a tax increase is overshifted if $dp/dt_s > d\phi/dt_s$. Part (a) and (b) can be verified by inspecting equation (8), remembering that $L(\bar{\theta}) = 0$ and $L(\theta) > 0 \forall \theta \in [\theta^*, \bar{\theta})$.

The effect on the per unit charge of a marginal increase in the ad valorem tax rate is

$$\frac{dp}{dt_v} = \phi q_p \frac{1}{\pi_{pp}} = \phi \frac{dp}{dt_s} > 0.$$

An ad valorem tax is overshifted if $dp/dt_v > p$, that is if, $\phi(dp/dt_s) > p$. Since $p > \phi$ an ad valorem tax is less likely to be overshifted. ■

C Proof of Proposition 2

The total derivative of A with respect to t_s is

$$\begin{aligned} \frac{dA}{dt_s} &= v_p(p(\theta, t_s, t_v), \theta) \frac{dp}{dt_s} + v_\theta(p(\theta^*, t_s, t_v), \theta^*) \frac{d\theta^*}{dt_s} \\ &\quad - \int_{\theta^*}^{\theta} v_{\theta p}(p(u, t_s, t_v), u) \frac{dp}{dt_s} du \end{aligned}$$

Notice that the term $v_\theta(p(\theta^*, t_s, t_v), \theta^*) \frac{d\theta^*}{dt_s}$ is zero if either $\theta^* > \underline{\theta}$, in which case, v_θ approaches zero, or $\theta^* < \underline{\theta}$, in which case, $\frac{d\theta^*}{dt_s}$ is zero. The incidence reduces to equation (11).

Part (b) of Proposition 2 is proved by the following. Another way of expressing the incidence term follows by adding the term $q(p(\theta, t_s, t_v), \theta) - q(p(\theta^*, t_s, t_v), \theta^*) -$

$\int_{\theta^*}^{\theta} q_p(p(u, t_s, t_v), u) \frac{dp}{d\theta} du - \int_{\theta^*}^{\theta} q_{\theta}(p(u, t_s, t_v), u) du = 0$. We get:

$$\begin{aligned} \frac{dA}{dt_s} &= q(p(\theta, t_s, t_v), \theta) \left(1 - \frac{dp}{dt_s}\right) - q(p(\theta^*, t_s, t_v), \theta^*) - \\ &\quad \int_{\theta^*}^{\theta} q_{\theta}(p(u, t_s, t_v), u) \left(1 - \frac{dp}{dt_s}\right) du - \int_{\theta^*}^{\theta} q_p(p(u, t_s, t_v), u) \frac{dp}{d\theta} du, \end{aligned}$$

When undershifting occurs following a change in t_s , it follows that dA/dt_s is negative at $\bar{\theta}$. When t_s is overshifted the sign of dA/dt_s cannot be determined in general.

Part (c) is proved by inspection of (14), from which it is seen that the right hand side approaches zero as ε_p and q_{pp} approaches zero. Hence, the change in the fixed fee following a tax increase is nonincreasing in θ for linear demand. Further, when $\frac{dp}{dt_s} \leq \frac{1}{2}$ we must have that $\frac{q_{p\theta}}{q_{pp}} < 0$. Because this ensures $\frac{d^2 p}{dt_s d\theta} = \frac{q_{pp}}{\pi_{pp}} \left[\frac{dp}{d\theta} (2 \frac{dp}{dt_s} - 1) - \frac{q_{p\theta}}{q_{pp}} \right] > 0$, this suffices to prove that $\frac{d^2 A}{dt_s d\theta} < 0$.

To confirm the last part of Proposition 2 we just need $\frac{E}{-\varepsilon_p}$ to be sufficiently large to satisfy equation (14). ■

D Calculations for Proposition 3

Here, we present all the calculations required to derive equations (15) and (16), which are used to prove Proposition 3.

$$\begin{aligned} R_V^{UP} - R_S^{UP} &= \int_{\theta^{**}}^{\bar{\theta}} \left\{ t_v(p^{UP}(t_s, t_v)q(p^{UP}(t_s, t_v), \theta) - t_s q(p^{UP}(t_s, t_v), \theta)) \right\} f(\theta) d\theta, \\ &= t_v \int_{\theta^{**}}^{\bar{\theta}} \left\{ p^{UP}(t_s, t_v)q(p^{UP}(t_s, t_v), \theta) - \frac{t_s}{t_v} q(p^{UP}(t_s, t_v), \theta) \right\} f(\theta) d\theta, \\ &= t_v \int_{\theta^{**}}^{\bar{\theta}} \left(p^{UP}(t_s, t_v) - \frac{c}{1-t_v} \right) q(p^{UP}(t_s, t_v), \theta) f(\theta) d\theta = t_v \Pi^{UP}. \end{aligned}$$

where $\theta \in [\theta^{**}, \bar{\theta}]$ is the firm's market coverage under uniform pricing, and $p^{UP}(t_s, t_v)$ is the uniform price that maximizes profit.

$$\begin{aligned}
R_V^{TP} - R_S^{TP} &= \int_{\theta^*}^{\bar{\theta}} \left\{ t_v (p(\theta, t_s, t_v) q(p(\theta), \theta) + A(\theta, t_s, t_v)) - t_s q(p(\theta, t_s, t_v), \theta) \right\} f(\theta) d\theta, \\
&= t_v \int_{\theta^*}^{\bar{\theta}} \left\{ p(\theta, t_s, t_v) q(p(\theta, t_s, t_v), \theta) + A(\theta) - \frac{t_s}{t_v} q(p(\theta, t_s, t_v), \theta) \right\} f(\theta) d\theta, \\
&= t_v \int_{\theta^*}^{\bar{\theta}} \left(p(\theta, t_s, t_v) + \frac{A(\theta, t_s, t_v)}{q(p(\theta, t_s, t_v), \theta)} - \frac{c}{1 - t_v} \right) q(p(\theta, t_s, t_v), \theta) f(\theta) d\theta = t_v \Pi^{TP}.
\end{aligned}$$

E Proof of Proposition 4

The claims in Proposition 4 can be verified by totally differentiating the endogenous variables with respect to t_s and t_v , and by applying the tax reform rule in (17), i.e., $\bar{p} dt_v = -dt_s$. That is, we total differentiate the equations $p = p(\theta, t_s, t_v)$, $A = A(\theta, t_s, t_v)$, $\Pi = \Pi(t_s, t_v)$, $CS = CS(t_s, t_v)$, and $\theta^* = \theta^*(t_s, t_v)$.

The effect on the per usage fee and on the fixed fee is given by

$$\begin{aligned}
dp &= \frac{dp}{dt_v} dt_v + \frac{dp}{dt_s} dt_s = \phi \frac{dp}{dt_s} dt_v - \bar{p} \frac{dp}{dt_s} dt_v = -[\bar{p} - \phi] \frac{dp}{dt_s} dt_v < 0, \\
dA &= -[\bar{p} - \phi] \frac{dA}{dt_s} dt_v > 0.
\end{aligned}$$

The signs are determined by Propositions 1 and 2 including the qualifying assumptions for this proposition. The remaining effects are

$$\begin{aligned}
d\theta^* &= -[\bar{p} - \phi] \frac{d\theta^*}{dt_s} dt_v < 0, & dCS &= -[\bar{p} - \phi] \frac{dCS}{dt_s} dt_v > 0, \\
dK &= -[\bar{p} - \phi] \frac{dK}{dt_s} dt_v > 0, & d\Pi &= 0.
\end{aligned}$$

The cutoff type is described by the equation (7), $q(p(\theta^*, t_s, t_v), \theta^*) = 0$. Differentiating this implicitly with respect to θ^* and t_s yields

$$\left(q_p \frac{dp}{d\theta} + q_\theta \right) d\theta^* + q_p \frac{dp}{dt_s} dt_s = 0 \Rightarrow \frac{d\theta^*}{dt_s} = -q_p \frac{\frac{dp}{dt_s}}{q_p \frac{dp}{d\theta} + q_\theta} \geq 0$$

and with respect to θ^* and t_v yields

$$\left(q_p \frac{dp}{d\theta} + q_\theta\right) d\theta^* + q_p \phi \frac{dp}{dt_s} dt_v = 0 \Rightarrow \frac{d\theta^*}{dt_v} = -\phi q_p \frac{\frac{dp}{dt_s}}{q_p \frac{dp}{d\theta} + q_\theta} \geq 0$$

Aggregate consumer surplus is

$$CS(t_s, t_v) = \int_{\theta^*}^{\bar{\theta}} \left\{ \int_{\theta^*}^{\theta} v_\theta(p(u, t_s, t_v), u) du \right\} f(\theta) d\theta = \int_{\theta^*}^{\bar{\theta}} v_\theta(p(\theta, t_s, t_v), \theta) (1 - F(\theta)) d\theta,$$

and

$$\begin{aligned} \frac{dCS}{dt_s} &= - \int_{\theta^*}^{\bar{\theta}} q_\theta(p(\theta, t_s, t_v), \theta) \frac{dp}{dt_s} (1 - F(\theta)) d\theta - v_\theta(p(\theta^*, t_s, t_v), \theta^*) (1 - F(\theta^*)) \frac{d\theta^*}{dt_s} \\ &= - \int_{\theta^*}^{\bar{\theta}} q_\theta(p(\theta, t_s, t_v), \theta) \frac{dp}{dt_s} (1 - F(\theta)) d\theta < 0, \end{aligned}$$

$$\frac{dCS}{dt_v} = \phi \frac{dCS}{dt_s} < 0.$$

Aggregate consumer expenditure is given by

$$\begin{aligned} K(t_s, t_v) &= \int_{\theta^*}^{\bar{\theta}} \left(p(\theta, t_s, t_v) q(p(\theta, t_s, t_v), \theta) + A(\theta, t_s, t_v) \right) f(\theta) d\theta \\ &= \int_{\theta^*}^{\bar{\theta}} \left(p(\theta, t_s, t_v) q(p(\theta, t_s, t_v), \theta) + v(p(\theta, t_s, t_v), \theta) - \right. \\ &\quad \left. \frac{1-F(\theta)}{f(\theta)} v_\theta(p(\theta, t_s, t_v), \theta) \right) f(\theta) d\theta \end{aligned}$$

and

$$\begin{aligned}
\frac{dK}{dt_s} &= \int_{\theta^*}^{\bar{\theta}} \left(p(\theta, t_s, t_v) q_p(p(\theta, t_s, t_v), \theta) - \frac{1 - F(\theta)}{f(\theta)} v_{\theta p}(p(\theta, t_s, t_v), \theta) \right) \frac{dp}{dt_s} f(\theta) d\theta - \\
&\quad \left(p(\theta^*, t_s, t_v) q(p(\theta^*, t_s, t_v), \theta^*) + A(\theta^*, t_s, t_v) \right) \frac{d\theta^*}{dt_s} \\
&= \int_{\theta^*}^{\bar{\theta}} \phi q_p(p(\theta, t_s, t_v), \theta) \frac{dp}{dt_s} f(\theta) d\theta - \\
&\quad \left(p(\theta^*, t_s, t_v) q(p(\theta^*, t_s, t_v), \theta^*) + A(\theta^*, t_s, t_v) \right) \frac{d\theta^*}{dt_s} < 0, \\
\frac{dK}{dt_v} &= \phi \frac{dK}{dt_s} < 0.
\end{aligned}$$

To derive this expression, we have used the first-order condition.

The isolated effect of a change in the taxes on the firm's profit $\Pi(t_s, t_v)$ is given by

$$\frac{d\Pi}{dt_s} = \int_{\theta^*}^{\bar{\theta}} \left(\frac{\partial \pi}{\partial t_s} + \frac{\partial \pi}{\partial p} \frac{dp}{dt_s} \right) f(\theta) d\theta - \pi(p(\theta^*, t_s, t_v), \theta^*, t_s, t_v) \frac{d\theta^*}{dt_s},$$

The term $\pi(p(\theta^*, t_s, t_v), \theta^*, t_s, t_v) \frac{d\theta^*}{dt_s}$ is zero if either $\theta^* > \underline{\theta}$, in which case, $\pi(p(\theta^*, t_s, t_v), \theta^*, t_s, t_v)$ is zero, or $\theta^* < \underline{\theta}$, in which case, $\frac{d\theta^*}{dt_s}$ is zero. Using this information together with the foc $\partial \pi / \partial p = 0$ we can write

$$\frac{d\Pi}{dt_s} = - \int_{\theta^*}^{\bar{\theta}} q(p(\theta, t_s, t_v), \theta) f(\theta) d\theta < 0,$$

$$\frac{d\Pi}{dt_v} = - \int_{\theta^*}^{\bar{\theta}} (p(\theta, t_s, t_v) q(p(\theta, t_s, t_v), \theta) + A(\theta, t_s, t_v)) f(\theta) d\theta = \bar{p} \frac{d\Pi}{dt_s}$$

where

$$\bar{p} \equiv \frac{\int_{\theta^*}^{\bar{\theta}} (p(\theta, t_s, t_v) q(p(\theta, t_s, t_v), \theta) + A(\theta, t_s, t_v)) f(\theta) d\theta}{\int_{\theta^*}^{\bar{\theta}} q(p(\theta, t_s, t_v), \theta) f(\theta) d\theta}$$

Because consumers' aggregate expenditures increase, it follows that tax revenues increase. ■

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